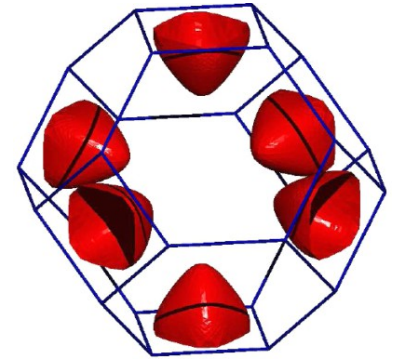


$$G = G_0 + G_0 V G$$



## ***First principles* calculations of electronic structure and physical properties**

*superconducting, thermoelectric, magnetic, magnetocaloric behaviors  
in intermetallic compounds with chemical disorder*

*Katedra Fizyki Materii Skondensowanej WFIS AGH (D10, III p.)*

*Grupa obliczeń struktur elektronowych*

prof. S. Kaprzyk, prof. J. Tobała

dr K. Kutorasiński, dr B. Wiendlocha

mgr P. Zwoleński (doktorant)

# Multiply scattering theory vs. Korringa-Kohn-Rostoker (KKR) method

Computational geometry

$$G = G_0 + G_0 V G. \quad \text{Dyson equations}$$

Crystal potential

$$V = \sum_i v_i.$$

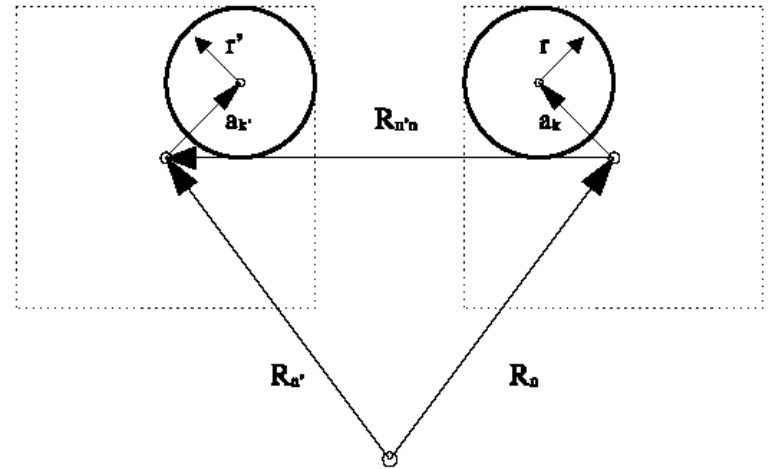
Free-electron GF

$$G_0 = \frac{1}{E - H}$$

WS cell

$$\mathbf{R}_{n'} = \mathbf{R}_n - \mathbf{R}_n$$

MT spheres



$T$  - scattering operator for many potentials

$$T = V + V G_0 T, \quad T = \sum_{i,k} \left[ v_i \delta_{ik} + v_i \frac{1}{E - H_0 - v_i} \sum_{m \neq i} (v_m \delta_{mk} + v_m \frac{1}{E - H_0 - V} v_k) \right]$$

$$T = \sum_{i,k} T^{i,k},$$

$T^{i,k}$  – path scattering operator between  $i$  and  $k$  scattering centres

$$T^{i,k} = t_i \delta_{ik} + t_i G_0 \sum_{m \neq i} T^{m,k},$$

$t$  - scattering operator for single potential  $v_i$

$$t_i = v_i + v_i G_0 t_i.$$

# KKR for ordered compounds

## Approaches

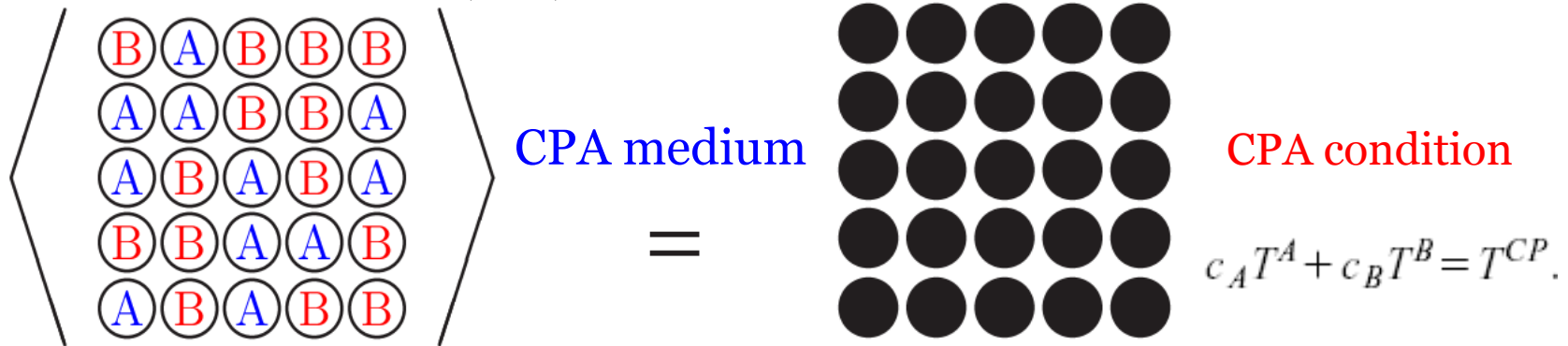
- \* Reasonable computational time for systems with unit cell below 100 atoms.
- \* Full form of crystal potential (no restricted to spherical); LDA(+U).
- \* Semi-relativistic approach (core electrons – relativistic, valence electrons – nonrelativistic).
- \* Fully-relativistic approach (all electrons from Dirac eq).
- \* Magnetic effects (spin & orbital contributions) accounting for different magnetic moments arrangements.

## Output

- \* Density of states (total &  $s$ ,  $p$ ,  $d$ , ... for inequivalent atoms).
- \* Electronic dispersion curves & Fermi surface.
- \* Magnetic moments (total & inside Voronoi polyhedra; spin & orbital) for different magnetic structures.
- \* Total energy of compound (and all contributions).

# KKR-CPA method & complex bands

Disordered alloys: ~~periodic~~ - Coherent Potential Approximation (CPA):



In multi-atomic systems more complicated !

$$T_{k'\sigma'L',k\sigma L}^{CP} = \frac{1}{N} \sum_{\mathbf{k} \in BZ} [\tau_{CP}^{-1} - B(E, \mathbf{k})]_{k'\sigma'L',k\sigma L}^{-1}$$

**CPA crystal - restored periodicity**

• **price: complex potential**

Present version of fully self-consistent KKR-CPA code allows for:

- treatment of **many atoms** on the same disordered site (< 60 atoms per unit cell).
- calculation of **electronic structure of dopants** (very low concentration),
- calculation of **extra point defects** in crystals (vacancy, anti-site, etc.)
- calculation of **spin dependent electronic structure** (magnetism).

# KKR-CPA for disordered alloys

## Approaches

- \* Reasonable computational time for systems with unit cell below 60 atoms.
- \* *Muffin-tin* form of crystal potential (LDA).
- \* Semi-relativistic approach (core electrons – relativistic, valence electrons – nonrelativistic).
- \* Spin-dependent effects taken into account.

## Output

- \* Density of states (total &  $s$ ,  $p$ ,  $d$ , ... for inequivalent atoms).
- \* Electronic dispersion curves for alloys (complex energy).
- \* Magnetic properties (total & inside MT spheres).
- \* Total energy of alloy (and all contributions).

# KKR & KKR-CPA strategies

## Crystal structure

- atoms distribution in the unit cell of alloys
- concentration of elements on crystallographic sites
- possible point defects if available
- dilatation/contraction of cell to mimic temperature effects

## KKR-CPA

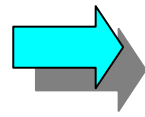
**Fast** Rough determination of ground state properties of  $A_x B_y C_{1-x-y}$  (metal/semiconductor; magnetic/non-magnetic) & their evolution with alloy composition

**Medium** Attempts to improve requested properties of new materials  $A_x B_y C_{1-x-y}$  via substitution with novel elements  $D$

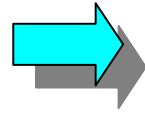
**Slow** More sophisticated analysis of physical properties (Fermi surface properties governing electron transport, electron-phonon coupling, different magnetic orderings of magnetic moments); **role of doping**

Attempts to establish „electronic phase diagrams” for ternary or pseudo-quaternary systems, tending to predict (or **not to miss**) new properties.

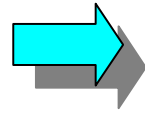
# Tematy prac doktorskich 2015



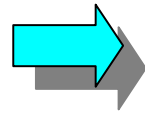
Badanie struktury elektronowej i oddziaływania elektron-fonon w nadprzewodzących izolatorach topologicznych (*lub inna grupa materiałów*); *J. Tobała & B. Wiendlocha*



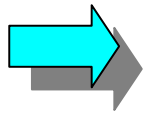
Implementacja nowych potencjałów wymiennie-korelacyjnych w metodzie KKR-CPA dla układów złożonych; *J. Tobała & B. Wiendlocha*



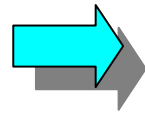
Teoretyczne poszukiwania nowych stopów termoelektrycznych i optymalizacja układów znanych *J. Tobała & K. Kutorasiński*



Rola efektów relatywistycznych w materiałach do konwersji termoelektrycznej i magnetokalorycznej; *J. Tobała*



Obliczenia wpływu entropii na stabilność krystaliczną, strukturę elektronową i własności magnetyczne stopów wieloatomowych *J. Tobała*



Struktura elektronowa i własności transportowe nowych materiałów na ogniwa jonowe; *J. Tobała*

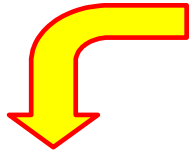
# Badanie struktury elektronowej i oddziaływania elektron-fonon w nadprzewodzących izolatorach topologicznych



**Electron-phonon interaction:** source of classical superconductivity

Important parameter: **electron-phonon coupling constant**  $\lambda$  0.1 (Cu),  $\sim 1$  (Nb),  
 $\sim 1.5$  (Nb3Sn)

Critical temperature: McMillan's equation:  $T_c = \frac{\omega_{log}}{1.20} \exp \left[ -\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right]$   $\mu^* \sim 0.10$   
-  
0.15



**task: calculate  $\lambda$**

**RMTA - rigid muffin tin approximation**

$$\lambda_{ep} = \sum_i \frac{\eta_i}{M_i \langle \omega_i^2 \rangle}$$

$\eta$  = response from electrons on atomic vibration

$$\eta_i = \sum_l \frac{(2l + 2)n_l^i(E_F)n_{l+1}^i(E_F)}{(2l + 1)(2l + 3)N(E_F)} \left| \int_0^{R_{MT}^i} r^2 R_l^i(E_F, r) \frac{dV^i(r)}{dr} R_{l+1}^i(E_F, r) \right|^2$$

**Electronic contribution**

**M  $\langle \omega^2 \rangle$  - lattice 'stiffness', phonon contribution**

$$\langle \omega^2 \rangle = \int_0^{\omega_{max}} \omega F(\omega) d\omega \Big/ \int_0^{\omega_{max}} \frac{F(\omega)}{\omega} d\omega$$

**F( $\omega$ ) - phonon density of states**



# Implementacja nowych potencjałów wymiennie-korelacyjnych w metodzie KKR-CPA dla układów złożonych

## Density Functional Theory

How to approximate  $E_{xc}$  (LDA -local density approximation) ?

$$E_{xc}[n] = \int d^3r n(\mathbf{r}) \epsilon_{xc}(n(\mathbf{r}))$$

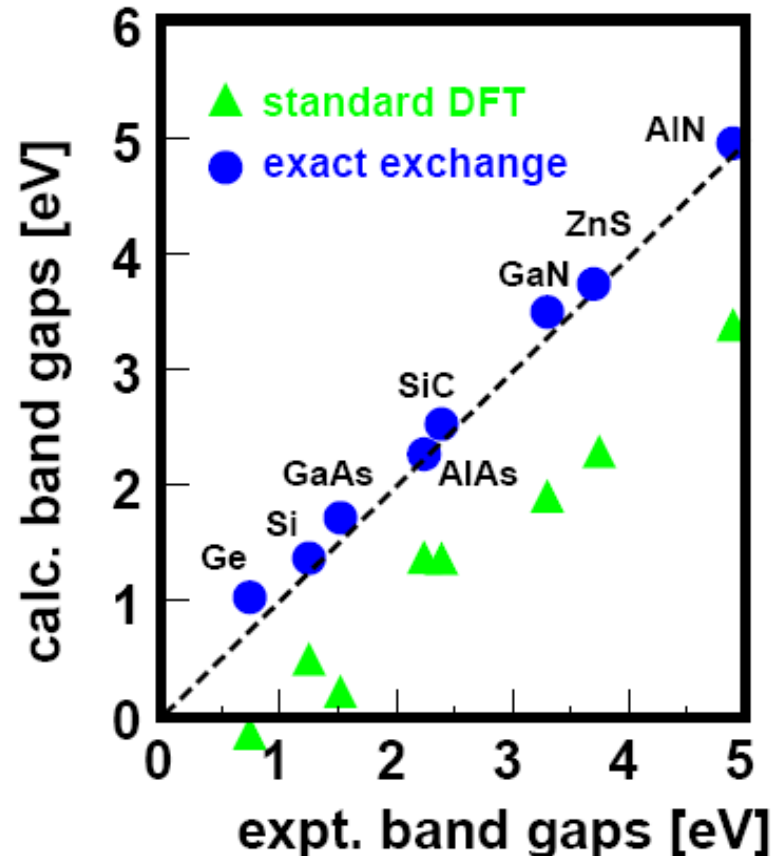
There are many analytical expressions for  $E_{xc}$ :

- Barth-Hedin, Vosko-Nusair, Lunqvist,
- Perdew-Wang, ...(1970-80),
- Car-Parinello (QMC)

LSD -local spin density approximation

GGA -generalised gradient approx.

$$E_{xc}^{GGA}[n^\uparrow, n^\downarrow] = \int d^3r n(\mathbf{r}) \epsilon_{xc}(n^\uparrow, n^\downarrow, \nabla n^\uparrow, \nabla n^\downarrow)$$
$$\equiv \int d^3r n(\mathbf{r}) \epsilon_x^{hom}(n) F_{xc}(n^\uparrow, n^\downarrow, \nabla n^\uparrow, \nabla n^\downarrow),$$



# Thermoelectric properties

## search for optimum

Carnot limit

Improvement of figure of merit

COOLING ELEMENTS

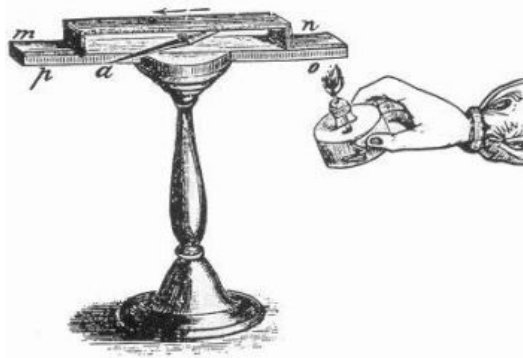
$$COP = (T_H - T_C)(\gamma - 1)(T_C + \gamma T_H)^{-1}$$

POWER GENERATORS

$$\eta = (\gamma T_C - T_H)[(T_H - T_C + (\gamma + 1))]^{-1}$$



Geometry of the devices



Physical properties of the system  $\gamma = (1 + ZT)^{1/2}$



$$ZT = \frac{S^2 \sigma}{\kappa} T = \frac{S^2}{\underbrace{L}_{\text{calculated}}} \frac{1}{1 + \frac{\kappa_L}{\kappa_e}}$$

$$L \equiv \frac{\kappa_e}{T \sigma}$$

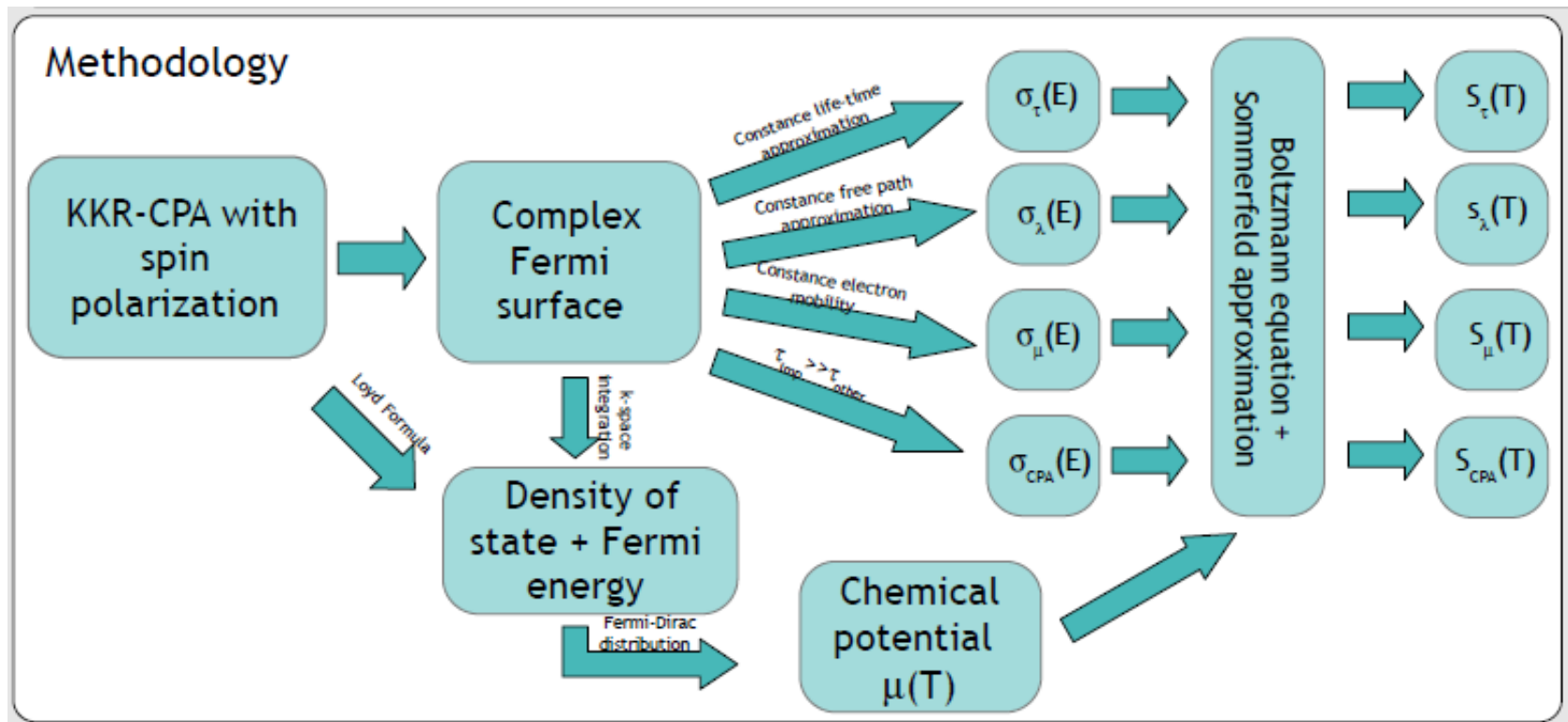
Lorentz factor

Thermal conductivity  
(phonons / electrons)

# Teoretyczne poszukiwania nowych stopów termoelektrycznych i optymalizacja układów znanych

K. Kutorasinski (grant NCN, Sonata) **2 stypendia** 2000zł/mies. przez 3 lata (2015-2018); preferencje: 1 fizyk + 1 informatyk

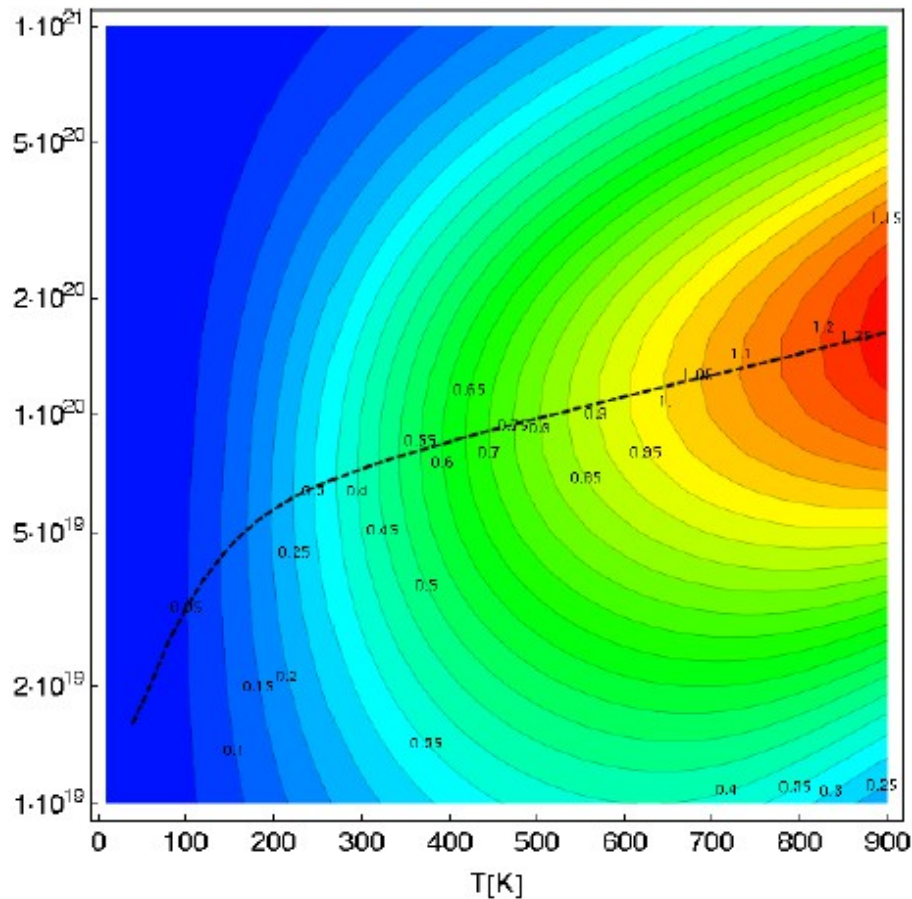
## Boltzmann transport & KKR-CPA calculations



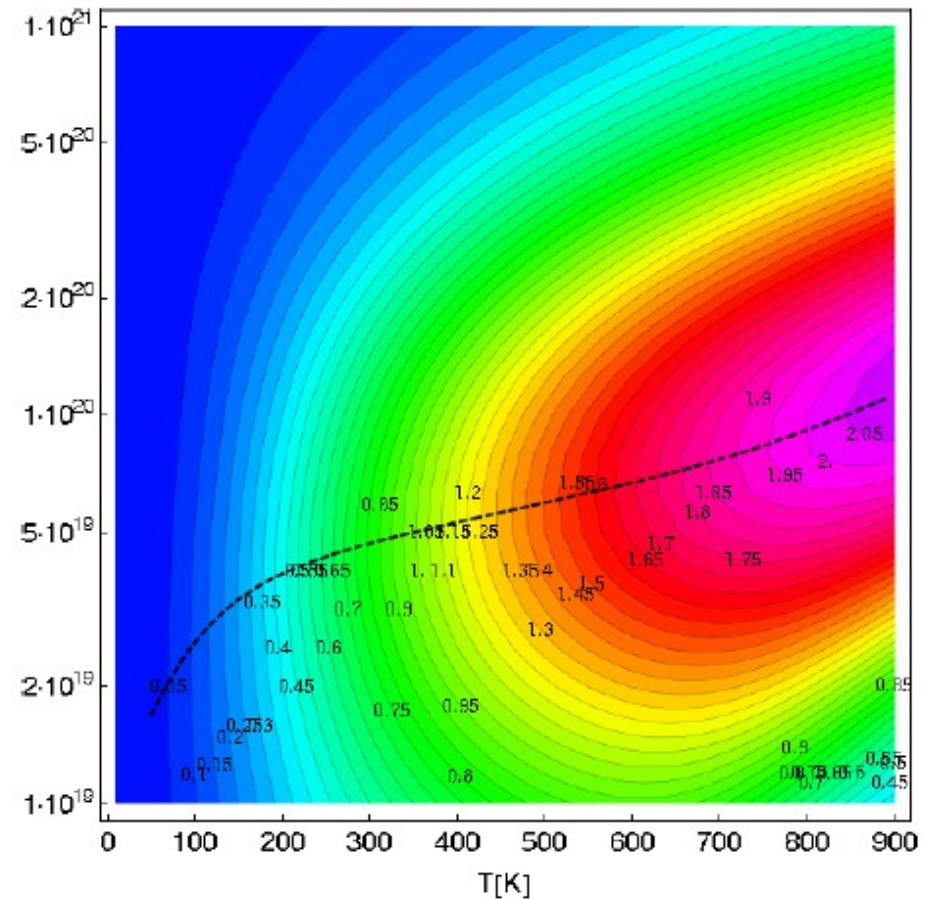
- (1)  $\tau = \text{const}$ ; (2)  $\lambda = \text{const}$ ; (3)  $\mu = \text{const}$ ; (4) CPA (velocity + life-time);

# ZT vs. n/p & T

lattice thermal conductivity as parameter



$\kappa_l = 1.25 \text{ W/mK}$



$0.5 \text{ W/mK}$

Kutorasinski et al. (JT), Phys. Rev. B 87 (2013) 195205.

Full form of Dirac equation including four components

$$\left( \beta mc^2 + c(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3) \right) \psi(x, t) = i\hbar \frac{\partial \psi(x, t)}{\partial t}$$

$$\begin{pmatrix} (mc^2 - E + e\phi) & c\sigma \cdot (p - \frac{e}{c}A) \\ -c\sigma \cdot (p - \frac{e}{c}A) & (mc^2 + E - e\phi) \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

More readable: non-relativistic approach of Dirac equation

$$H = H_0 + H_{\text{kinetic}} + H_{\text{so}} + H_{\text{Darwinian}}$$

$$H_{\text{so}} = \frac{1}{2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right) \left( \frac{g_s}{2m_e^2 c^2} \right) \frac{\vec{L} \cdot \vec{S}}{r^3}$$

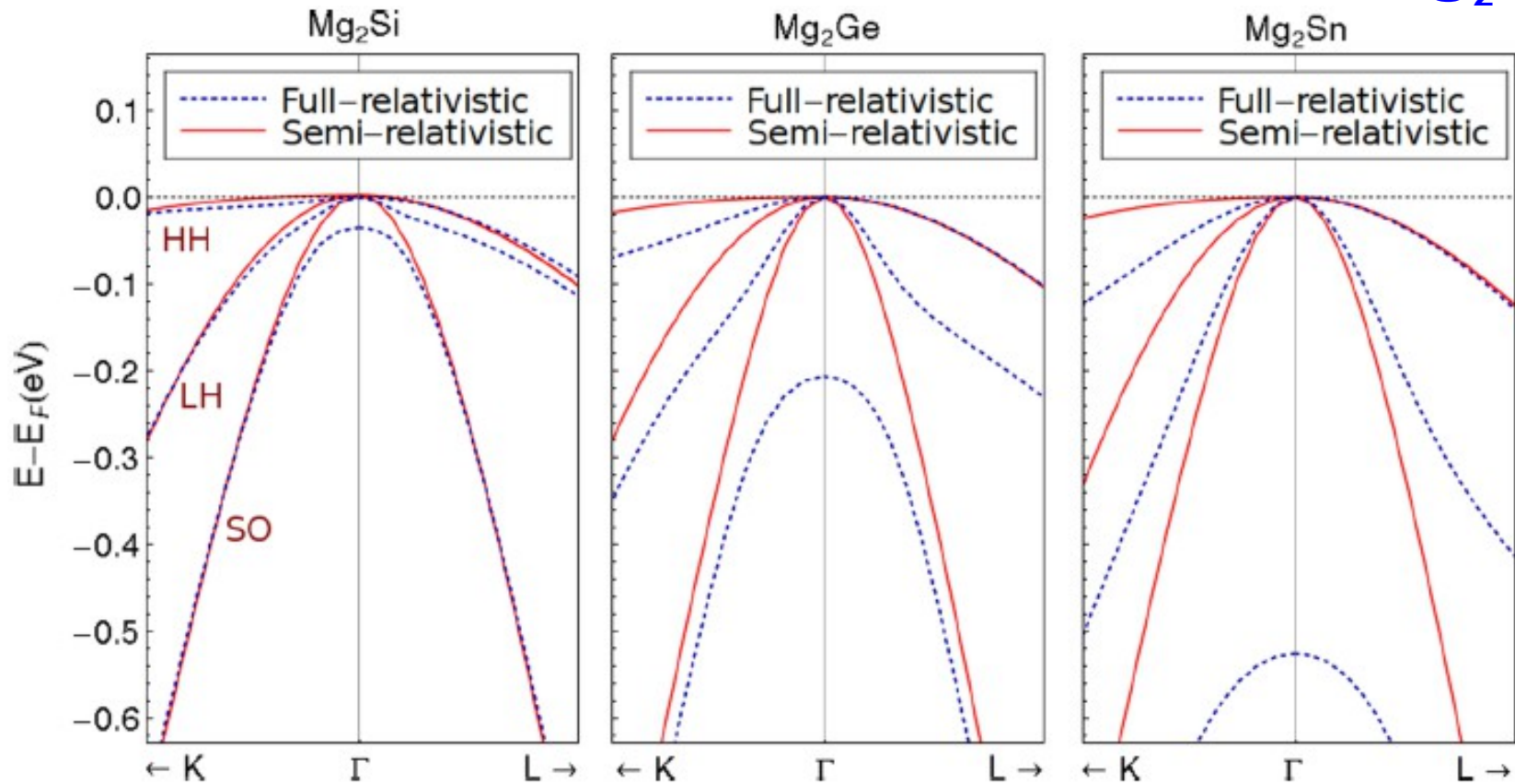
$$H_{\text{kinetic}} = -\frac{p^4}{8m^3 c^2}$$

$$\langle H_{\text{SO}} \rangle = \frac{E_n^2}{m_e c^2} \left( n \frac{j(j+1) - l(l+1) - \frac{3}{4}}{l(l+\frac{1}{2})(l+1)} \right)$$

$$H_{\text{Darwinian}} = \frac{\hbar^2}{8m_e^2 c^2} 4\pi \left( \frac{Ze^2}{4\pi\epsilon_0} \right) \delta^3(\vec{r})$$

$$\text{SO splitting} \sim \frac{Z^4}{n^3(j+1/2)} 10^{-5} \text{ eV}$$

# Effect of S-O on electronic bands in $Mg_2X$

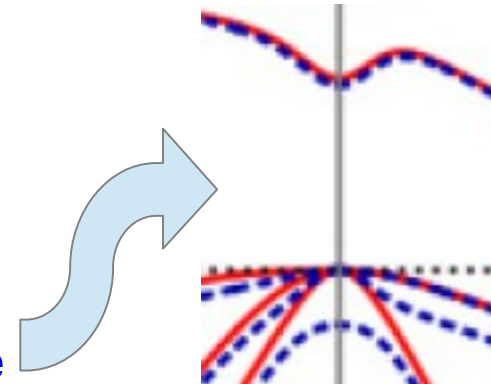


|                  | $X=$ | Si        | Ge         | Sn                   |
|------------------|------|-----------|------------|----------------------|
| Calculated (meV) |      | 36        | 298        | 544                  |
| Measured (meV)   |      | $30^{11}$ | $200^{11}$ | $480^{11}, 600^{12}$ |

<sup>11</sup> F. Vazquez, A. R. Forman, and M. Cardonna *Phys. Rev.*, vol. 176, p. 905, 1968

<sup>12</sup> L. A. Lott and D. W. Lynch, *Phys. Rev.* 141, 681 (1965)

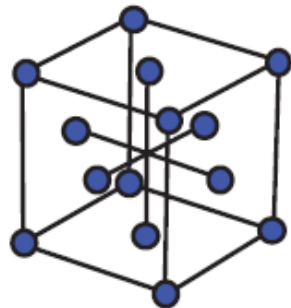
effect of S-O: on conduction bands - negligible  
 on valence bands - HUGE!



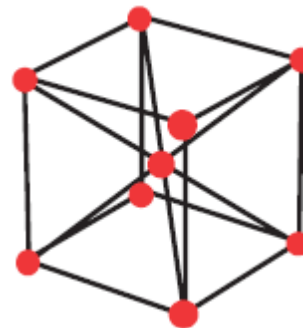
# Obliczenia wpływu entropii na stabilność krystaliczną, strukturę elektronową i własności magnetyczne stopów wieloatomowych

## Many factors important:

- electronegativities of elements,
- VEC number of valence electrons
- atomic coordinations,
- symmetry of atoms sites,
- magnetic interactions,
- **degree of chemical disorder**



$$F = E - TS$$



NONG Zhi-sheng, et al/Trans. Nonferrous Met. Soc. China 22(2012) 1437-1444

## FeTiCoNiVCrMnCuAl 系高熵合金中金属间化合物的第一性原理计算

农智升<sup>1,2</sup>, 朱景川<sup>1,2</sup>, 于海玲<sup>1</sup>, 来忠红<sup>1</sup>

1. 哈尔滨工业大学 材料科学系, 哈尔滨 150001;

2. 哈尔滨工业大学 金属精密热加工国家级重点实验室, 哈尔滨 150000

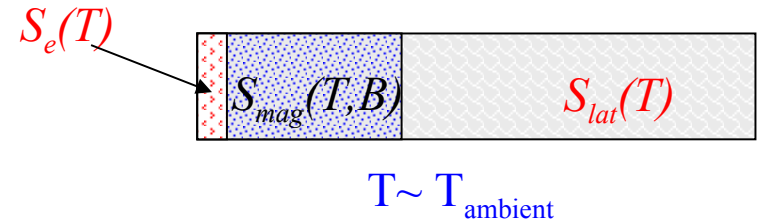
| No. elements<br>$N$ | Configurational<br>Entropy |
|---------------------|----------------------------|
| 2                   | 0.693                      |
| 3                   | 1.098                      |
| 4                   | 1.386                      |
| 5                   | 1.609                      |
| 6                   | 1.791                      |
| 7                   | 1.945                      |
| 8                   | 2.079                      |
| 10                  | 2.302                      |

$$S_{conf} = -k_B \sum_{i=1}^N c_i \ln c_i \quad \text{with} \quad \sum_{i=1}^N c_i = 1$$

## High entropy alloys

$$S_{conf} = -k_B \sum_{i=1}^N 1/N \ln 1/N = k_B \ln N$$

# Magnetocaloric effect & MC materials



## Entropy

$$S_{\text{tot}} = S_{\text{el}} + S_{\text{mag}} + S_{\text{lat}}$$

$$\Delta S_{\text{mag}} \uparrow + \Delta S_{\text{lat}} \downarrow = 0$$

$$\Delta S_{\text{mag}}(T, \Delta B) = \int_0^B \left( \frac{\partial M}{\partial T} \right)_B . dB$$

## Adiabatic Process

Calorific Capacity

$$C_p(B, T)$$

Magnetic Entropy

$$\Delta S_{\text{mag}}$$

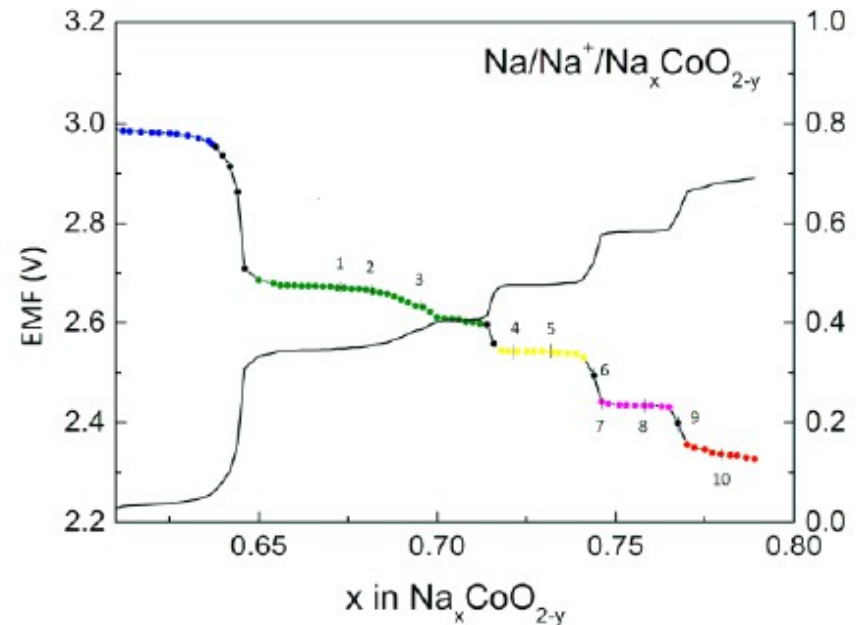
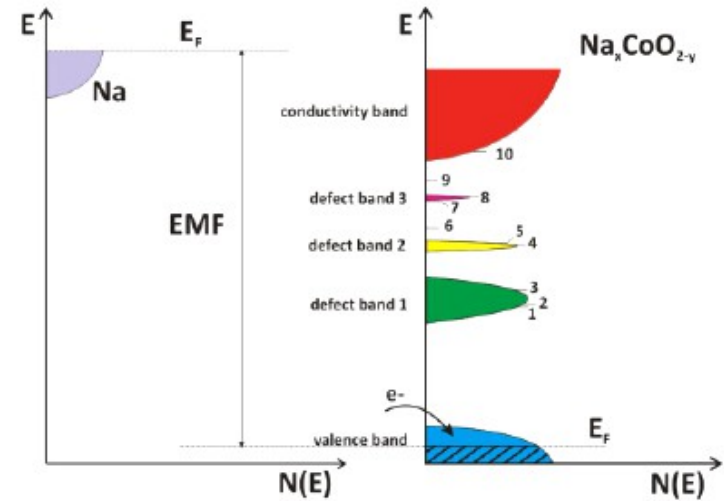
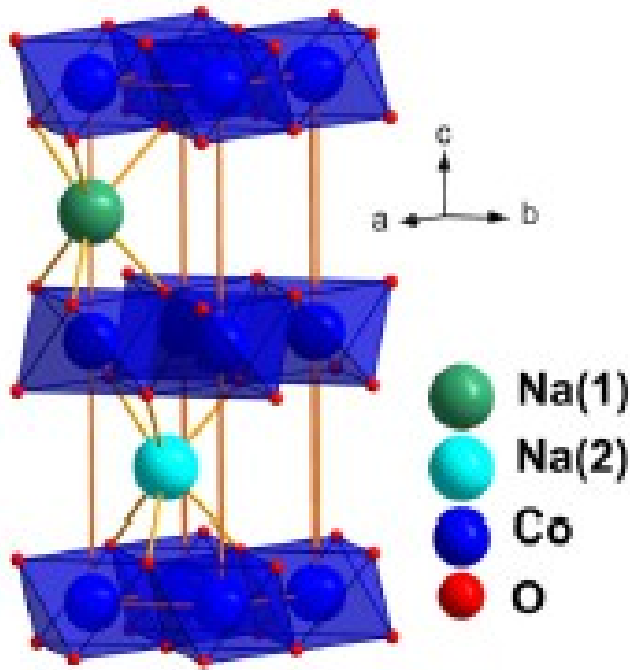
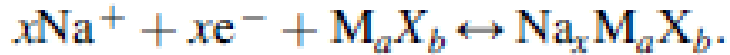
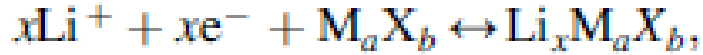
$$\Delta T_{\text{max}}$$

$$\Delta S_{\text{lat}} = C_p(B, T) \frac{\Delta T}{T}$$

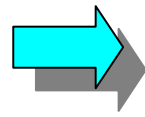
$$\Delta T_{\text{max}} = \frac{-T \cdot \Delta S_{\text{mag}}}{C_p(B, T)}$$



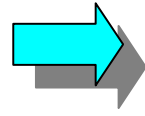
# Struktura elektronowa i własności transportowe nowych materiałów na ogniwa jonowe



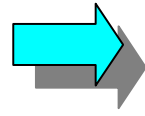
# Tematy prac doktorskich 2015



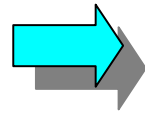
Badanie struktury elektronowej i oddziaływania elektron-fonon w nadprzewodzących izolatorach topologicznych (*lub inna grupa materiałów*); *J. Tobała & B. Wiendlocha*



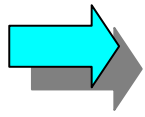
Implementacja nowych potencjałów wymiennie-korelacyjnych w metodzie KKR-CPA dla układów złożonych; *J. Tobała & B. Wiendlocha*



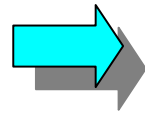
Teoretyczne poszukiwania nowych stopów termoelektrycznych i optymalizacja układów znanych *J. Tobała & K. Kutorasiński*



Rola efektów relatywistycznych w materiałach do konwersji termoelektrycznej i magnetokalorycznej; *J. Tobała*



Obliczenia wpływu entropii na stabilność krystaliczną, strukturę elektronową i własności magnetyczne stopów wieloatomowych *J. Tobała*



Struktura elektronowa i własności transportowe nowych materiałów na ogniwa jonowe; *J. Tobała*