Spin-orbit coupling anisotropy and avoided-crossings in energy spectra of a quantum dot in an in-plane magnetic field

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Experimentally resolved energy spectra of an InAs quantum dot in external magnetic field exhibits avoided crossings that are induced by the spin-orbit coupling. The width of the avoided crossings depends strongly on the orientation of the magnetic field which reveals the intrinsic anisotropy of the spin-orbit interactions. We use exact diagonalization approach to study dependence of avoided crossing width on the orientation of the magnetic field and explain the physics standing behind the dependence.

Introduction

The spin-orbit interaction (SOI) has been receiving growing interest which is motivated by the possibility of the all-electrical control of the electron spin which is beneficial for many spintronics applications.

In quantum dots SOI induces mixing between electronic states which manifests itself as an avoided crossings (AC) between energy levels. Recent excited-state spectroscopy measurements probed those AC, directly measuring the strength of SOI in a single [1] and a double quantum dot [2] defined in quantum wires or in a gated self-organized quantum dot [3]. The latter experiment revealed the dependence of the AC width on the orientation of external magnetic field. The purpose of the present work is to explain this anisotropic proprieties of an elongated quantum dot in the presence of the external magnetic field and Rashba and/or Dresselhaus SOI [4].

Theory

Two electron problem described by the Hamiltonian:

$$H = h_1 + h_2 + \frac{e^2}{4\pi\varepsilon\varepsilon_0|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{1}$$

Single electron Hamiltonian :

$$h = \left(\frac{\hbar^2\mathbf{k}^2}{2m^*} + V(\mathbf{r})\right)\mathbf{1} + \frac{1}{2}g\mu_B\mathbf{B}\cdot\sigma + H_{BIA} + H_{SIA}, \tag{2}$$

Rashba SOI term , due to asymmetry of the structure:

$$H_{SIA} = \alpha\left[\frac{\partial V}{\partial z}\right](\sigma_{x'}k_{y'} - \sigma_{y'}k_{x'}). \tag{3}$$

The slope of the potential due to electric field in the growth direction (e.g. resulting from backgate voltage)

Dresselhaus SOI term , due to asymmetry of the crystal host:

$$H_{BIA} = \gamma\left[\sigma_{x'}k_{x'}(k_z^2 - k_{y'}^2) + \sigma_{y'}k_{y'}(k_{x'}^2 - k_z^2) + \sigma_z k_z(k_{y'}^2 - k_{x'}^2)\right], \tag{4}$$

with strength dependent on the geometry of the structure (e.g. height of the dot).

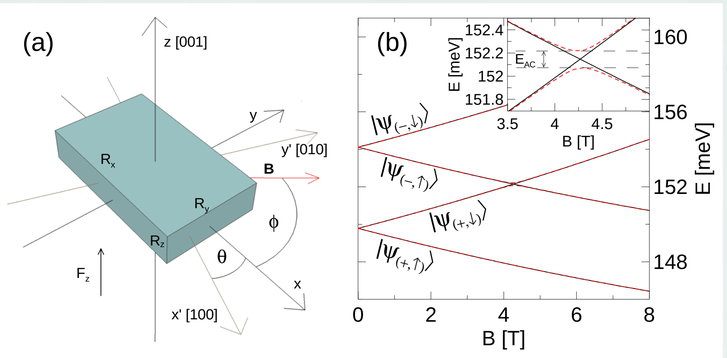


Figure 1. *(a) Schematic of the quantum dot with coordinate system used. (b) One-electron energy spectrum with Dresselhaus interaction present (red lines) and absent (black lines) for magnetic field oriented along y-direction.*

The system : three-dimensional quantum-dot with limited height, $R_x = 100nm, R_y = 60nm$ and $R_z = 10nm$, described by the potential $V(x, y, z) = V(x) + V(y) + V(z)$. Parameters taken for InAs. The x', y' - axes correspond with [100] and [010] crystal directions respectively, x, y and z axes corresponds with the axes of the dot.

The method : solution of the three-dimensional two-electron problem in few steps:

- separation of the three dimensional Hamiltonian (2) into parts that are separable into x, y, z-directions and a part that is non-separable and spin-dependent
- solution of the 1D problems on grids
- diagonalization of the nonseparable spin-dependent part of the Hamiltonian in a basis of 1D solutions,
- configuration-interaction method with one-electron, 3D solutions to obtain two-electron spin-orbitals

Results

The width of the AC depends on the orientation of in-plane magnetic field in a different manner for Rashba and Dresselhaus coupling.

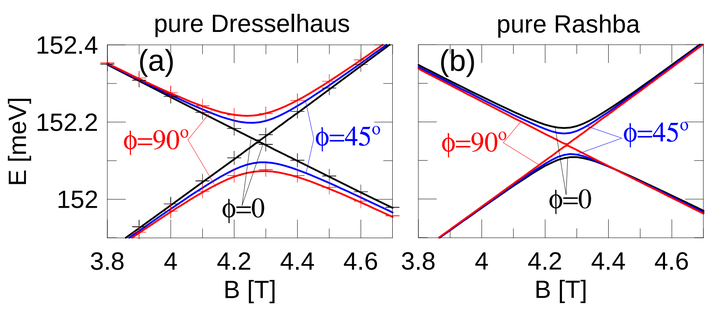


Figure 2. *Anticrossing width for different orientations of the in-plane magnetic field.*

The AC width depends on the ratio in which SOI mixes the states of opposite parity and spin, which can be evaluated considering matrix elements of SOI interaction Hamiltonian in a basis of those states. For Dresselhaus coupling with effective 2D Hamiltonian $H_{BIA}^{2D} = \gamma^{2D}\left(\sigma_x k_x - \sigma_y k_y\right)$.

- Magnetic field oriented along y-direction ($\phi = 90°$),

$$\begin{pmatrix} E_{(+,\downarrow)} + \gamma^{2D}\langle\Psi_{(+,\downarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(+,\downarrow)}\rangle & \gamma^{2D}\langle\Psi_{(+,\downarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(-,\uparrow)}\rangle \\ \gamma^{2D}\langle\Psi_{(-,\uparrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(+,\downarrow)}\rangle & E_{(-,\uparrow)} + \gamma^{2D}\langle\Psi_{(-,\uparrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(-,\uparrow)}\rangle \end{pmatrix}. \tag{5}$$

  The terms vanish due to their spin and parity.

$$\begin{pmatrix} E_{(+,\downarrow)} & -i\gamma^{2D}\langle\Psi_{(+,\downarrow)}|\sigma_x\frac{\partial}{\partial x}|\Psi_{(-,\uparrow)}\rangle \\ -i\gamma^{2D}\langle\Psi_{(-,\uparrow)}|\sigma_x\frac{\partial}{\partial x}|\Psi_{(+,\downarrow)}\rangle & E_{(-,\uparrow)} \end{pmatrix}. \tag{6}$$

  The non-vanishing off-diagonal matrix elements mix the states $|\Psi_{(+,\downarrow)}\rangle, |\Psi_{(-,\uparrow)}\rangle$ which results in an avoided crossing between the corresponding energy levels.
- Magnetic field aligned parallel to the x-direction ($\phi = 0$).

$$\begin{pmatrix} E_{(+,\leftarrow)} + \gamma^{2D}\langle\Psi_{(+,\leftarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(+,\leftarrow)}\rangle & \gamma^{2D}\langle\Psi_{(+,\leftarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(-,\rightarrow)}\rangle \\ \gamma^{2D}\langle\Psi_{(-,\rightarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(+,\leftarrow)}\rangle & E_{(-,\rightarrow)} + \gamma^{2D}\langle\Psi_{(-,\rightarrow)}|\sigma_x k_x - \sigma_y k_y|\Psi_{(-,\rightarrow)}\rangle \end{pmatrix}. \tag{7}$$

  All SOI dependent term vanish and energy levels of $|\Psi_{(+,\leftarrow)}\rangle, |\Psi_{(-,\rightarrow)}\rangle$ states cross.

The width of AC is periodic function of magnetic field angle. The position of the AC minimum depends on the dot orientation with respect to crystal host.

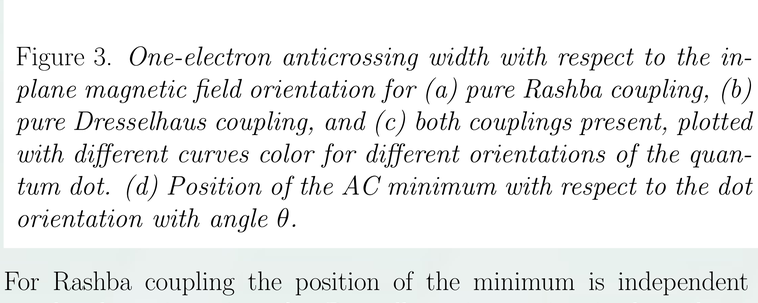
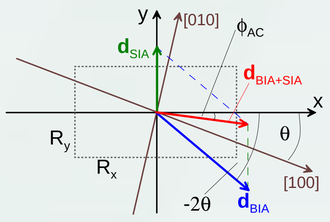


Figure 3. *One-electron anticrossing width with respect to the in-plane magnetic field orientation for (a) pure Rashba coupling, (b) pure Dresselhaus coupling, and (c) both couplings present, plotted with different curves color for different orientations of the quantum dot. (d) Position of the AC minimum with respect to the dot orientation with angle θ.*

For Rashba coupling the position of the minimum is independent on the dot orientation, for Dresselhaus interaction it depends as $\phi_{AC} = -2\theta$ and for both couplings present with strength ratio $\frac{\gamma^{2D}}{\alpha^*} = -\frac{\gamma}{\alpha}\frac{\pi^2}{|e|F_z R_z^2}$ it depends as

$$\tan(\phi_{AC}) = \frac{1+\frac{\gamma^{2D}}{\alpha^*}\sin(-2\theta)}{\frac{\gamma^{2D}}{\alpha^*}\cos(2\theta)},$$



In the two-electron regime AC appears in the lowest part of the energy spectrum – between energy levels of singlet and triplet states – it is accessible experimentally.

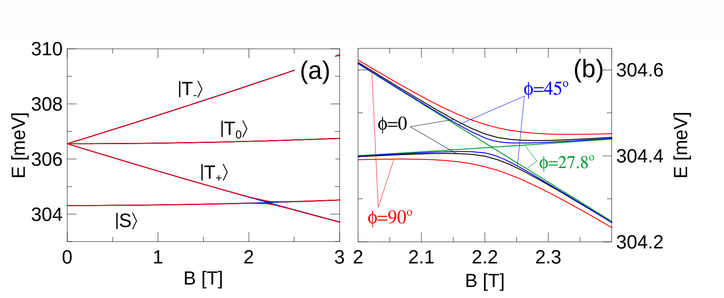


Figure 4. *(a) Energy spectrum of a two-electron quantum dot in the presence of Rashba and Dresselhaus interaction. (b) Zoom of the AC region for different orientations of the magnetic field.*

The AC width undergoes $|\sin(\phi - \phi_{AC})|$ dependence when the orientation of the magnetic field is changed.

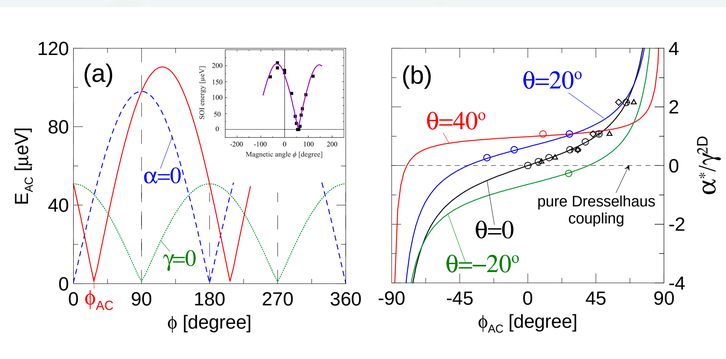


Figure 5. *(a) Anticrossing width in function of the magnetic field angle. In the inset: experimental results (dots) along with dependence obtained from our calculation (purple curve). (b) Position of the AC minimum for different strength of the SOI and orientations of the dot.*

The knowledge of the position of AC minimum and the orientation of the dot allows to evaluate relative strength of SO couplings trough the relation: $\frac{\alpha^*}{\gamma^{2D}} = \cos(2\theta)\left[\tan(\phi_{AC}) - \tan(-2\theta)\right]$.

Conclusions

- For anisotropic quantum-dot dependence of the AC width on the orientation of the magnetic field bears signatures of spin and parity mixing due to SOI.
- For pure Rashba SOI, AC is minimal when the magnetic field is aligned along the short axis of the dot.
- Dresselhaus SOI introduces dot-orientation-dependent shift of the position of the minimum.
- The AC width in function of magnetic field angle undergoes $\sin(\phi - \phi_{AC})$ dependence and shift ϕ_{AC} is influenced by relative strength of both SOI and the dot orientation.
- Experimental measurements of the dependence of AC in function of the in-plane orientation of the magnetic field can be used to extract the relative strengths of both SO interactions present in the nanostructure.

Acknowledgements

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