Statistics of spin avalanches in antiferromagnetic nano-networks

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Abstract

NETWORKS of nanoparticles with dipolar interactions are of interest for their new magnetic properties [1]. Such networks can be formed of cobalt [2], iron [3] or ferrihydrite [4] nanoparticles. We consider two types of networks, the scale-free (SC) and the exponential (EX) networks of size N with the number M of nodes to which new nodes are attached. Each node contains a nanoparticle with spin $S_i = \pm 1$. The dipolar interaction is limited to pairs of neighbours in the network. In our numerical experiment we simulate the hysteresis loop in $T = 0$. We investigate compact spin avalanches, using the damage spreading method (DS) [5]. The character of obtained avalanches from DS is different than in the conventional methods [6, 7]. The results indicate that the avalanche spectra are characterized by the same statistics as the degree distribution in their home networks. We calculate the range Z of avalanches, i.e. the distance between the damaged node and the furthest changed nodes. For small networks we obtained a new scaling relation between the range Z and the size s of avalanches.

$$Z \propto N^{\beta} f(N^{-\alpha} s) \tag{1}$$

where α and β are growing from 0 (for $M = 1$) to $\alpha = 0.5$ and $\beta = 0.33$ (for $4 < M < 10$). These values are the same for both investigated networks. A check for $M = 25$ confirms α and β values and the independence. Similar behaviour of α and β is found for the avalanche diameter.

1. Calculations

WE considered two types of growing networks: exponential (EX) and scale-free (SC) systems, where in each node we put on spin value $S_i = \pm 1$. The system was in the external magnetic field H, which maximum value we set as

$$H_{max} = k_{max} + \delta \tag{2}$$

where we put $\delta = 0.5$, and k_{max} was the maximum value of node degree in network. We were changing H from H_{max} to $-H_{max}$ with step $\delta_H = -1$. After each change of H, we were updating directions of spins in network until the energy of every node in network

$$E_i = \sum_j S_i S_j - H S_i \tag{3}$$

was minimal. Spins were updated in sequences of four orders: from oldest to newest nodes (A) or back (B), with random permutations (C) or with one given order (D). Local minimum of energy was found for each field.

In our calculations we used the damage spreading (DS) method [5] by introduction another network (U') with the same structure as original network (U) but with a damage. As the damage we set one spin in the opposite direction to the one it had before a change of the external magnetic field. Next we were changing H and when the energy got a local minimum we were comparing the state of the network with the damage to the state without it and we were counting the number of spins of different values.

All results were averaged over L networks. Each field change was performed N times to improve the statistics; in this way each node was used once for DS technique.

2. Results

THE results were obtained for the network size $N = 2 \cdot 10^3$. Figures 1 and 2 represent the average number of avalanches N_L against the avalanche size s for both types of networks. The statistics was obtained for $L = 10$ networks.

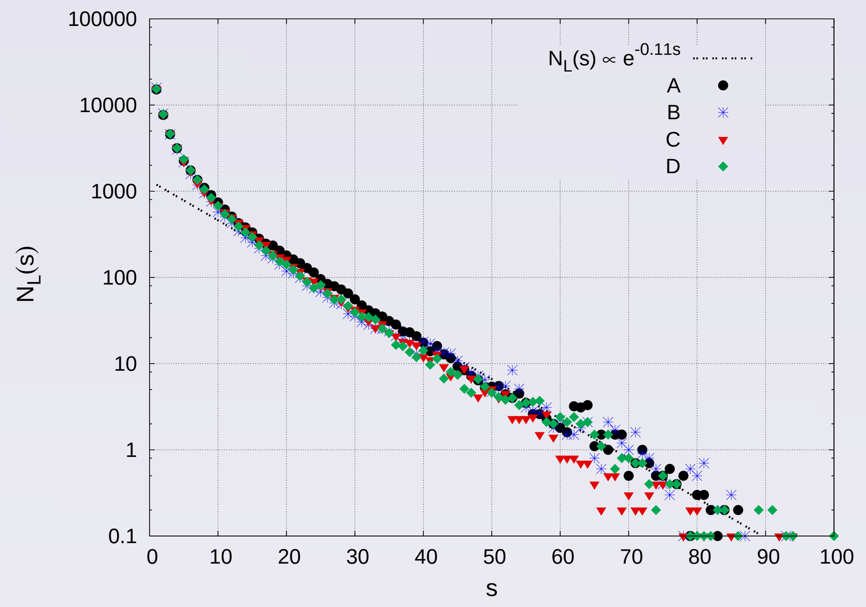

Figure 1: *Avalanches from DS method in the exponential networks, where $M = 5$.*

For the exponential networks the (A) method of updating spins was applied. We have got

$$N_L(s) \propto e^{-\phi s} \tag{4}$$

with $\phi = 0.45$ for $M = 1$, $\phi = 0.21$ for $M = 2$ and $\phi = 0.11$ for $M = 5$. For the scale-free network we obtained

$$N_L(s) \propto s^{-\gamma} \tag{5}$$

with $\gamma = 2.71$ for $M = 1$, $\gamma = 2.14$ for $M = 2$ and $\gamma = 1.86$ for $M = 5$.

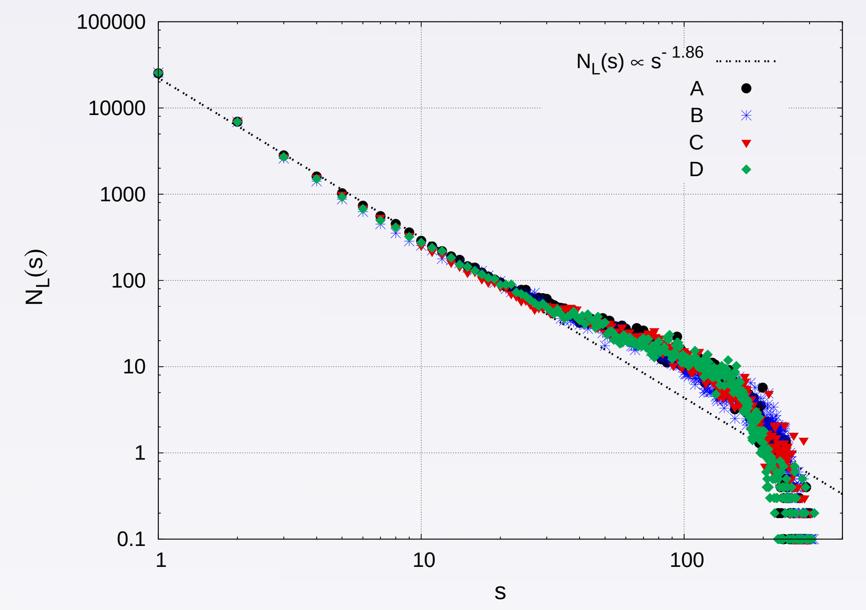

Figure 2: *Avalanches from DS method in scale-free networks, where $M = 5$.*

Using method (A) of updating spins we considered the maximal distance Z reached by an avalanche from the damaged site, against the avalanche size s. Results were averaged over $L = 10^2$ networks for $N = 300$ and over $L = 10^3$ for other values of N.
From our calculations we obtained the dependence (1), where $\alpha \approx 0.5$, and $\beta \approx 0.33$ for both kind of networks and $M \geq 4$.

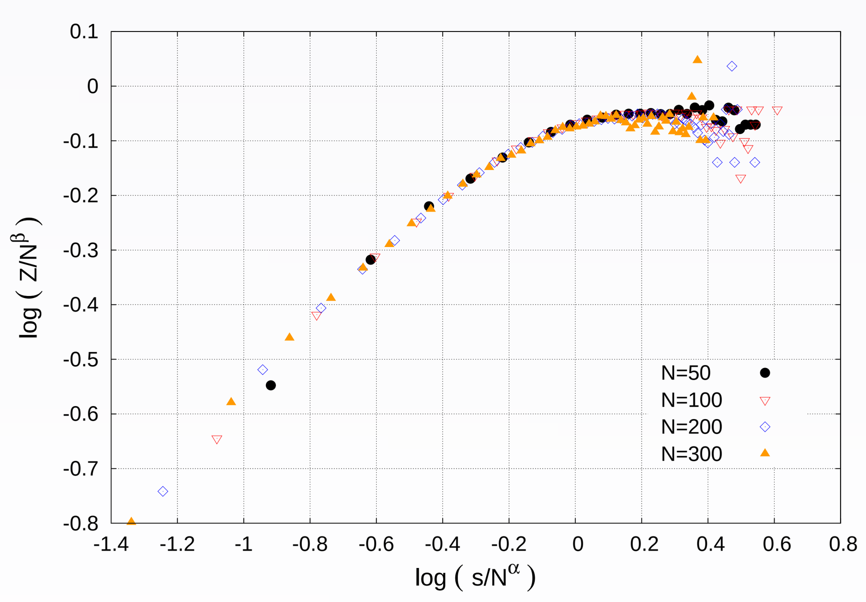

Figure 3: *The scaling relation for the exponential networks of size N, where $M = 5$.*

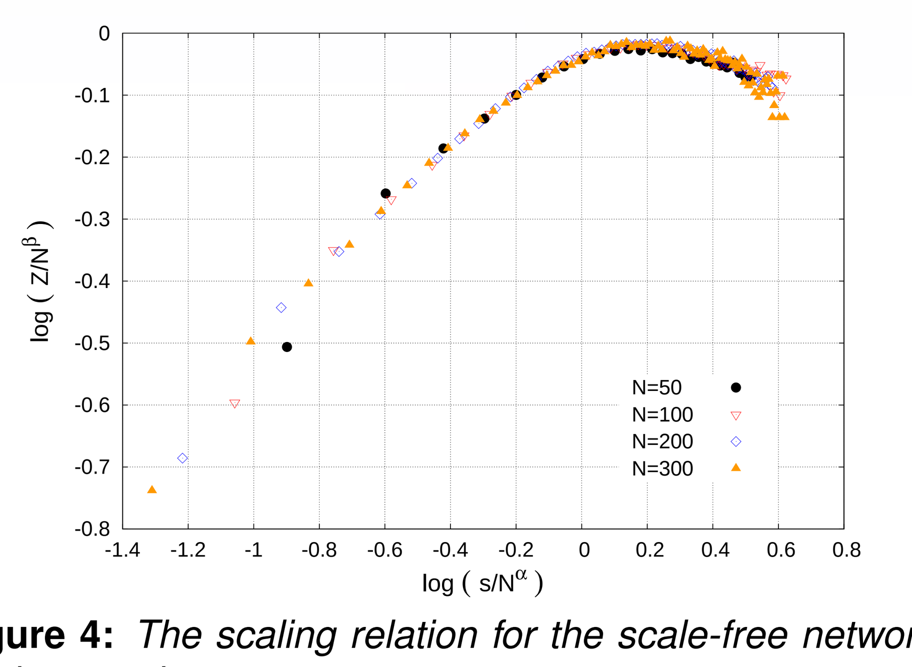

Figure 4: *The scaling relation for the scale-free networks of size N, where $M = 5$.*

For the exponential and scale-free trees ($M = 1$) we obtained $\alpha = 0$ and $\beta = 0$. These results were averaged over $L = 10^3$ networks for $N = 300$, and over $L = 10^4$ for other values of N.

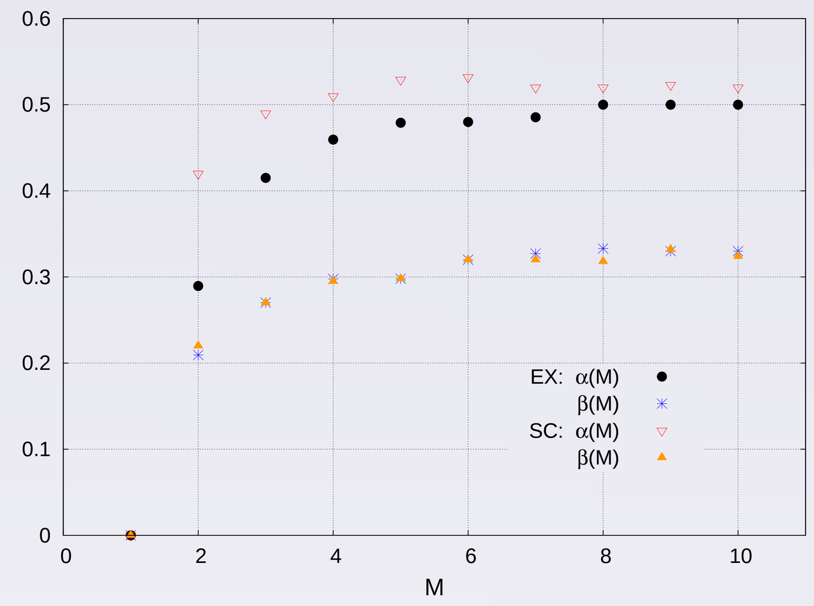

Figure 5: *The exponents α and β against M for the exponential (EX) and the scale-free networks (SC).*

For the mean avalanche diameter D the formula (1) also applies for the exponential networks ($M \geq 5$) with $\alpha \approx 0.5$, $\beta \approx 0.37$. For the scale-free networks: $\alpha \approx 0.54$ ($M \geq 5$) and β seems to tend to 0.40 ($M \geq 8$). For $M = 1$ we obtained $\alpha = 0$ and $\beta = 0$ in both kind of networks, in the same way as for the range Z.

3. Discussion

OUR numerical results indicate that the spectrum of avalanches is described by the same function as the degree distribution in the growing network. The observed coincidence indicates that there is a proportionality between the size of avalanche and the degree of the node at the avalanche origin. We checked that indeed this proportionality does appear.

The second result independent on the network topology is the scaling relation between the range and size of the avalanches, i.e. $Z/N^{\beta} = f(s/N^{\alpha})$, where $N_{max} = 300$. For trees ($M = 1$), the exponents α and β vanish, what reflect the fact that in linear chains the range and the size of avalanches is the same. For $M = 4$ and larger, α is close to 1/2 and β is close to 1/3 with the numerical accuracy. This is true both for the exponential and the scale-free networks. The plateau of the function $f(s)$, shown in Fig. 3, reveals that the range of avalanches is limited: The scaling relation found here suggests that the origin of this limitation is the size of the network. We are not aware of any analytical calculation of the range of avalanches. The topological disorder of the network, combined with the frustration, can produce a kind of magnetic disorder, analogous to the Random Field Model [8]; perhaps some generalization of this model could be applied here.

Acknowledgments

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