Purification of the spin-orbit coupling type by the Zeeman effect in quantum dots

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We study a single and two-electron parabolic quantum dots in the presence of Dresselhaus and Rashba spin-orbit interactions. We demonstrate that for negative Landè factors the effect of the Dresselhaus coupling is suppressed at high magnetic field, which for structures without inversion asymmetry leads to a completely spin-polarized system and a strict antisymmetry of the wave functions with respect to the interchange of spatial-electron coordinates.

Introduction

In quasi-two-dimensional semiconductor systems inversion asymmetry of the potential profile in the growth direction introduces spin-orbit (SO) interaction for the confined carriers known as structure-asymmetry-induced or Rashba coupling [1]. Moreover, in III-V's and II-VI's the inversion asymmetry of the crystal lattice introduces SO coupling of the Dresselhaus [2] type.

Coupling of the spin and orbital degrees of freedom is an important issue for quantum information processing applications based on spins of electrons confined in quantum dots [3]. The initial and final states of a quantum gate should preferably correspond to quantum-dot-confined stationary states with a definite spin orientation. However, the SO coupling leads to decay of the spin polarization [4]. For two-electron quantum dots this amounts in the triplet-singlet relaxation [5].

In transport experiments which probe the ground-state properties of confined systems, usually both types of SO ape present and the separation of their relative contribution is difficult [6]. The purpose of this work is to establish the role of both types of SO coupling in context of the ground-state spin-polarization at high magnetic field and the separability of spin and spatial degrees of freedom.

Theory

We consider the effective mass single-electron Hamiltonian of the form

$$H = h_0 + H_R + H_D \tag{1}$$

where h_0 stands for the Hamiltonian of the electron in a two-dimensional parabolic quantum dot for perpendicular magnetic-field B without the SO coupling

$$h_0 = \left(\frac{\mathbf{p}^2}{2m^*} + \frac{m^*\omega^2\mathbf{r}^2}{2} \right) \mathbf{1}, \tag{2}$$

with $\mathbf{p} = -i\hbar\nabla + e\mathbf{A}$, where $e > 0$ is the elementary charge. The symmetric gauge $\mathbf{A} = \frac{B}{2}(-y, x, 0)$, $m^* = 0.063m_0$ and $\hbar\omega = 2$ are applied.

In Eq. (1) $H_R(H_D)$ is the Rashba (Dresselhaus) coupling term

$$H_R = \frac{\alpha}{\hbar}(\sigma_x p_y - \sigma_y p_x), H_D = \frac{\beta}{\hbar}(\sigma_x p_x - \sigma_y p_y), \tag{3}$$

where α and β are the Rashba and Dresselhaus constants.

We solved the single-electron problem in the basis constructed of the eigenfunctions of the h_0 operator, i.e. the Fock–Darwin (FD) states.

For pure Rashba (Dresselhaus) coupling and confinement potential of circular symmetry, Hamiltonian (1) commutes with the total angular momentum operator $J_+ = L_z + S_z$ ($J_- = L_z - S_z$). For both types of SO coupling present we refer to the eigenstates of $P_s = P\sigma_z$ [P defined as $Pf(\mathbf{r}) = f(-\mathbf{r})$] corresponding to the eigenvalue +1(-1) as even (odd) s-parity states.

The two-electron Hamiltonian

$$H^{2e} = H(1) + H(2) + \frac{e^2}{4\pi\epsilon\epsilon_0 r_{12}}, \tag{4}$$

with GaAs-dielectric constant $\epsilon = 12.9$, is diagonalized in a basis of antisymmetrized products of the single-electron eigenfunctions of operator (1).

For the discussion of the spatial symmetry of the wave function, we consider an operator R which exchanges the spatial electron coordinates. Deviation of $|\langle R\rangle|$ from unity can be considered as a measure of the nonseparability of the spin and spatial degrees of freedom.

Results

In the absence of the Zeeman effect the energy spectra for pure Rashba and pure Dresselhaus couplings are identical. The energy of the second $J = -1/2$ state tends at high field to the FD ground state. The $J = 1/2$ ground-state energy level saturates at higher B and consequently the SO coupling energy tends to a constant in the high magnetic-field limit.

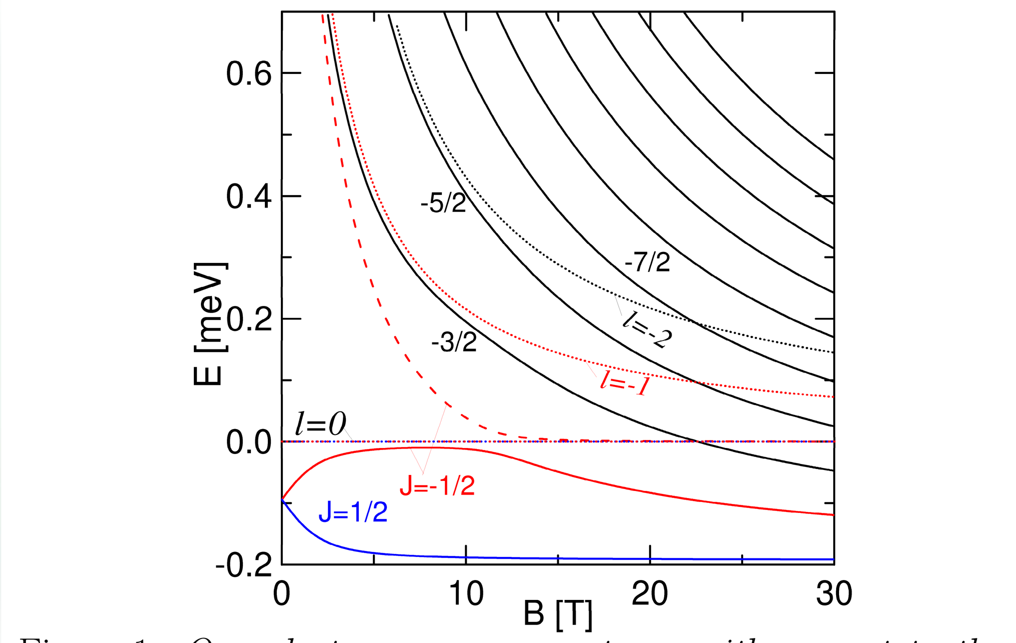

Figure 1. *One-electron energy spectrum with respect to the lowest FD level, calculated for $g = 0$. Only one type of SO coupling present, coupling constant of $c = 10.8meVnm$.*

For nonzero g at the high field the $J = -1/2$ ground-state energy becomes equal to the FD ground-energy level. The Zeeman effect at high magnetic field suppresses the Dresselhaus coupling induced mixing of the different spin states of the entire low-energy spectrum.

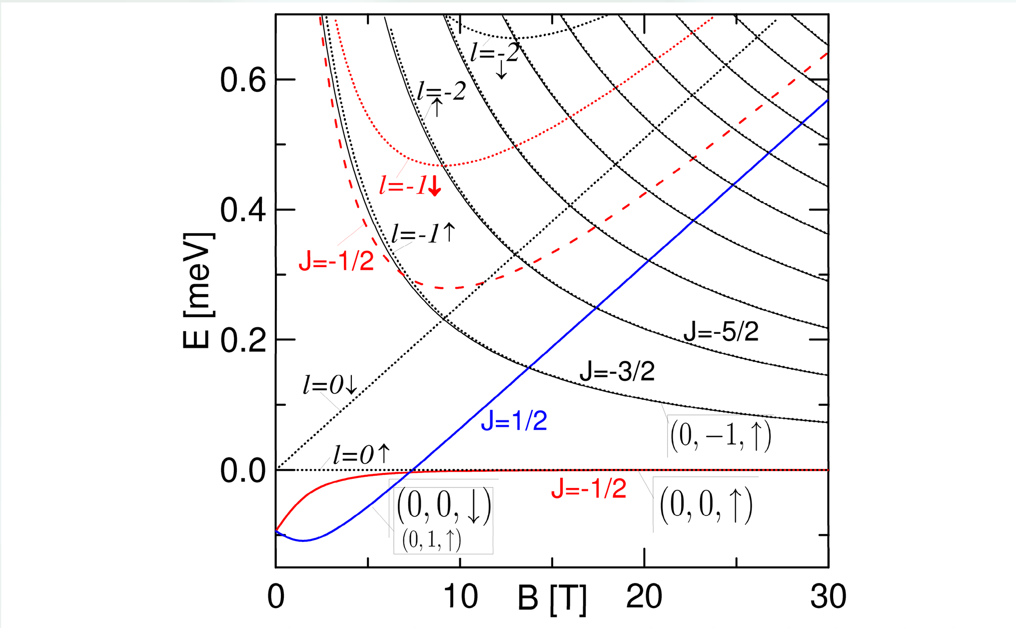

Figure 2. *One-electron lowest-energy levels calculated when only Dresselhaus coupling with $\beta = 10.8meVnm$ is present together with the Zeeman effect with $g = -0.44$.*

In contrast to the case with Dresselhaus case for pure Rashba coupling at high magnetic field the lowest-energy levels for a given J do not coincide with the FD spin-up energy levels (dotted lines in Fig. 3). This indicates that the lowest-energy states have nonvanishing spindown admixtures which preserve the presence of the SO energy shift.

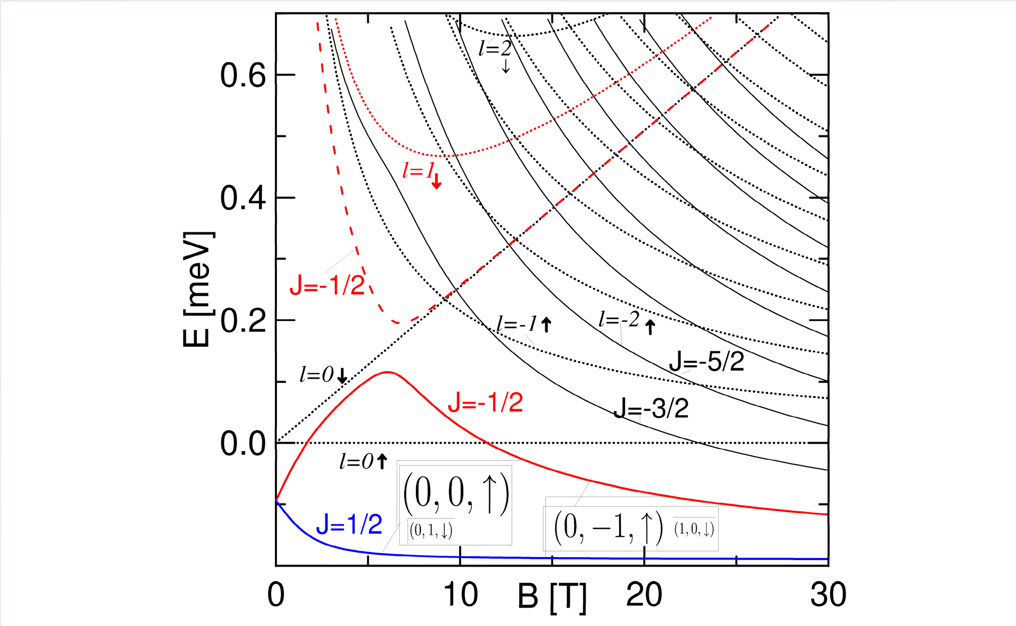

Figure 3. *Same as Fig.2 but for pure Rashba Coupling.*

For $\alpha = \beta = 10.8$ at high magnetic field the low-energy two-electron spectrum tends to the values obtained for pure Rashba coupling – effects of Dresselhaus coupling are suppressed.

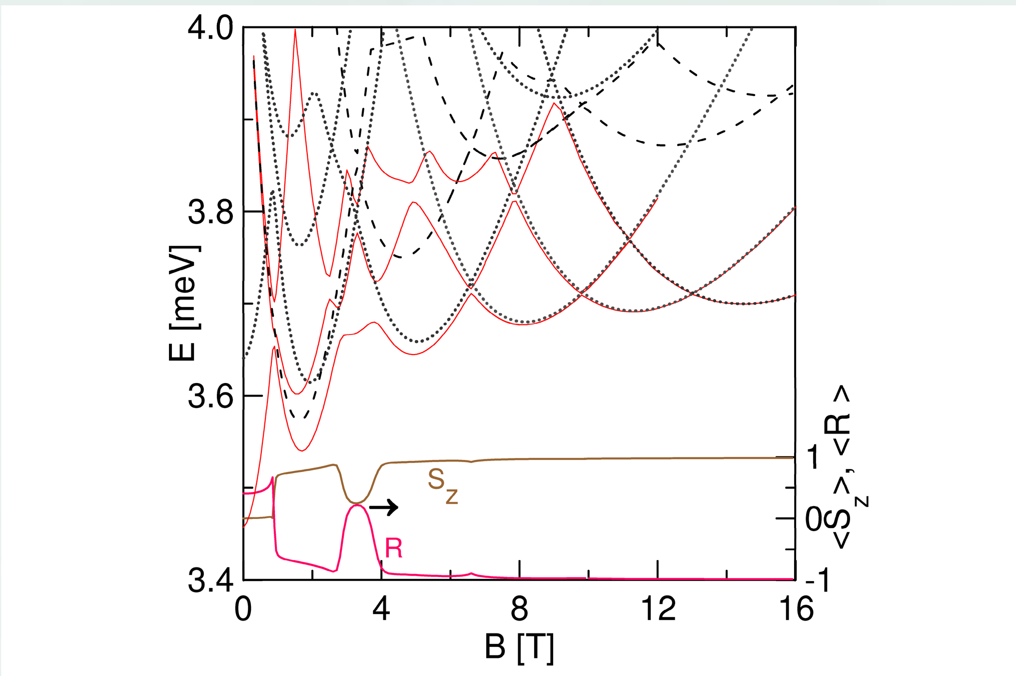

Figure 4. *Low-energy spectrum for two-electron states for $\alpha = \beta = 10.8meVnm$ and $g = -0.44$. Energy levels of odd (even) s parity states are plotted with solid red (black dashed) lines. Dotted lines show the energy levels obtained for pure Rashba coupling.*

Due to the removal of the Dresselhaus coupling by the strong field, the ground- state can be labelled by J_+ which is a good quantum number for pure Rashba coupling.

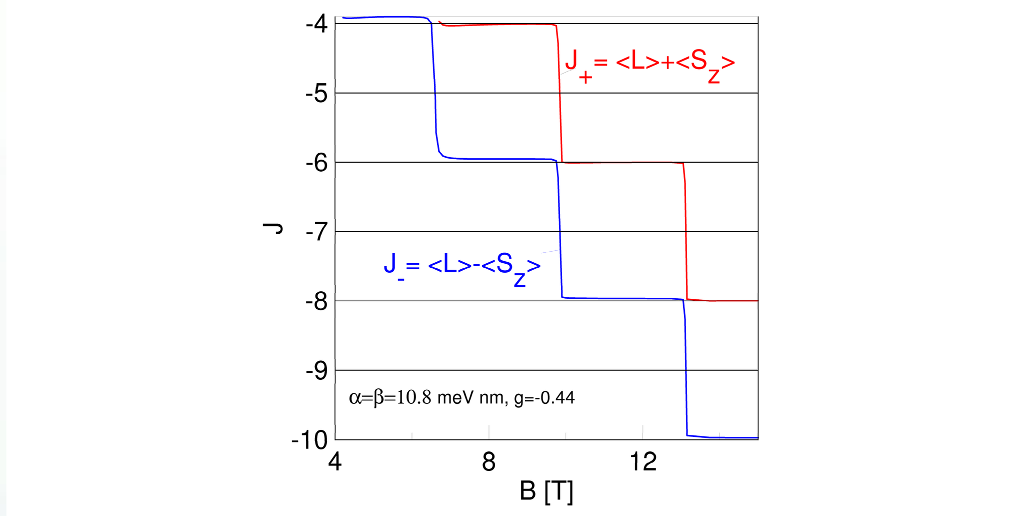

Figure 5. *The sum and the difference of the angular momentum and spin average value for two-electron ground-state.*

For nonzero values of α the spin-up polarization as well as antisymmetry of spatial wave function at high field is only approximate.

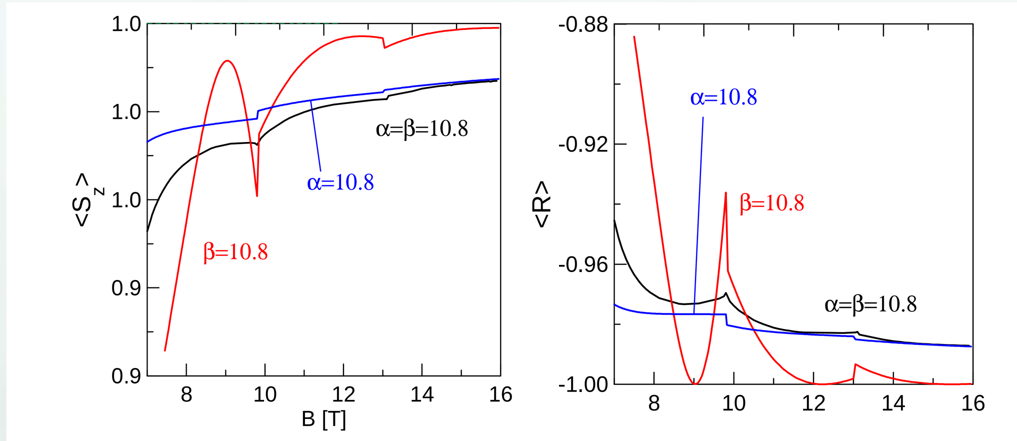

Figure 6. *Expectation values of the R and S_z operators in presence of the Zeeman effect $g = -0.44$.*

Conclusions

In the absence of the Zeeman effect the magnetic field does not suppress the SO coupling and the SO coupling energy saturates at high B. As a consequence the ground-state spin polarization is approximate but not complete. We demonstrated that Dresselhaus coupling effects are for $g < 0$ suppressed at high magnetic field. For $g < 0$ and both types of SO coupling present the low-energy spectrum at high B tends to the one obtained for pure Rashba coupling. The two-electron wave functions become only approximately antisymmetric with respect to exchange of the electron coordinates.

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